control the frequency of the most unstable disturbances, i.e., making their frequencies high enough to exceed the noisy environmental disturbance limit of existing wind tunnels. Calculations indicate that the windward meridian of a cooled, 7-deg half-angle cone at $\alpha = 4$ deg and a unit Reynolds number of 3.28 million per meter will result in a condition where most of the second-mode frequencies exceed the measurable disturbance frequency of the wind-tunnel freestream. Plans are being made to perform such experiments, and this will provide a unique opportunity to investigate hypersonic transition phenomena at a condition that is representative of what will be expected in flight. One important aspect of these forthcoming experiments will be to investigate the influence of a quiet environment on the nonlinear aspects of hypersonic boundary-layer transition and hopefully aid in determining the adequacy of linear stability theory for predicting hypersonic boundary-layer disturbance growth rates.

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Influence of a Favorable Pressure Gradient on the Growth of a Turbulent Spot

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Introduction

A RECENT overview of current knowledge on the turbulent spot was given by Riley and Gad-el-Hak.[1] Flow visualizations[2,3] and multipoint measurements of either velocity or temperature[4] have indicated that a spot contains several coherent structures within it. These structures have also been identified in a turbulent boundary layer,[5] and Bandyopadhyay[6] has noted several features that a turbulent spot and a turbulent boundary layer have in common. The present investigation represents part of a study whose long-term aim is to assess the influence of a pressure gradient on the organized motion of a turbulent boundary layer.

It seemed natural to focus first on the spot, since it provides a more controlled environment than a turbulent boundary layer. Wygnanski[7] noted that a favorable pressure gradient should slow down the growth process and, therefore, enable a more detailed analysis of transition than would be possible in the absence of a pressure gradient. The study of a favorable pressure gradient on the spot is also of practical value, since the leading edges of airfoils where the spots first form are regions of favorable pressure gradient and convex curvature, the effect of the latter being also stabilizing. The present Note, which considers the effect of a favorable pressure gradient on the overall three-dimensional development of a spot, complements and extends the study of Wygnanski.[7] Apart from comparing the three-dimensional growth rates with those obtained for zero pressure gradient, the present Note examines the effectiveness of different similarity coordinates in describing the global evolution of the spot with and without a pressure gradient. This latter aspect has not been previously addressed in the literature.

Experimental Arrangement and Conditions

The measurements were made in the working section of a low-speed, open-circuit-type blower wind tunnel of dimensions 5.4 m long, 0.89 m high, and 0.15 m wide when the walls are parallel. The position of one wall is adjustable to permit the pressure gradient to be easily changed. The aluminum plate over which the laminar boundary layer develops is vertical and is heated to about 10°K above the ambient temperature, i.e., $\Delta T = T_w - T_1 = 10°K$, where T_w and T_1 are the wall and freestream temperatures, respectively. As in the experiments of Antonia et al.,[4] the amount of heat introduced is small enough for temperature to act as a passive marker of the spot. The turbulent spot was triggered at a frequency of 1 Hz by a small jet of air issuing from a 3 mm diam hole, 300 mm from the contraction exit ($x = 0$), where the boundary-layer thickness is about 5 mm.

All measurements were made with a nominal reference freestream veloctiy U_s of 4.8 ms^{-1} at the location of the disturbance ($x_s = 0$). The hot wire (5 μm diam, Pt-10% Rh, 1.5 mm length) used for velocity measurements was operated with a DISA 55M10 constant temperature anemometer. For the temperature measurements, a single cold wire (0.6 μm diam, Pt-10% Rh, 1.0 mm length) was used with an in-house constant current (0.1 mA) circuit. The spot trigger signal and the cold wire signal were digitized at a sampling frequency of 3200 Hz and recorded on magnetic tape. Before introducing the spot, the quality of the laminar flow was checked by measuring the velocity and temperature profiles at several x stations for both zero and favorable pressure gradients. The pressure gradient dP/dx is related to the exponent m in the freestream velocity variation $U_1(x) = Kx^m$ by $m = -(x/\rho U_1^2)\, dP/dx$. For the pressure gradient considered here, $m = 0.05$. For both $m = 0$ and $m = 0.05$, reasonable agreement was found between measurement and the Blasius or Pohlhausen distributions.

Before calculating ensemble averages, the location of the leading edges (LE) and trailing edges (TE) of the spot were detected. (The terms LE and TE are commonly used in the spot literature and are retained here. LE and TE refer to the downstream and upstream boundaries, respectively, of the spot.) Since histograms of t_{LE} and t_{TE}, the leading and trailing edge arrival times relative to the spot trigger, indicated a larger variation in t_{LE} than in t_{TE}, t_{LE} was used in forming ensemble averages. To avoid spurious spots, only those spots with a t_{LE}

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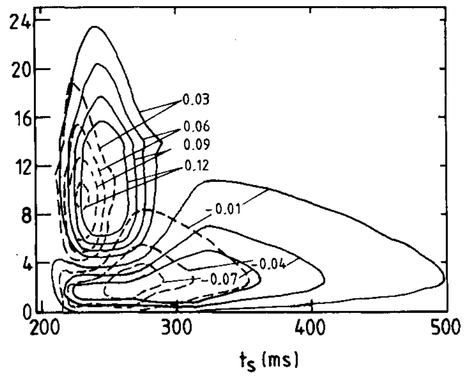

Fig. 1 Temperature disturbance contours in the (t_s, y) plane for $m = 0$ and $m = 0.05$; $x_s = 850$ mm, $z = 0$; ___, $m = 0$; _ _ _, $m = 0.05$.

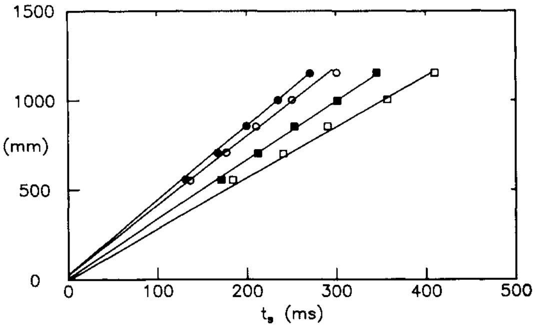

Fig. 2 Average arrival times of the leading edge and the trailing edge of the spot: $m = 0$: ○, leading edge; □, trailing edge; $m = 0.05$: ●, leading edge; ■, trailing edge.

falling within ±5% of the average t_{LE} were retained. Ensemble averages of the instantaneous temperature T were then determined for typically 120 spots in a manner similar to that of Antonia et al.[4]

Results and Discussion

Contours of constant $\tilde{T}/\Delta T$, where $\tilde{T}$ is the ensemble-averaged temperature fluctuation, have been obtained at five locations ($x_s = 550$, 700, 850, 1000, 1150 mm) in the plane of symmetry of the spot. Contours are shown in Fig. 1 for $x_s = 850$ mm, using y (distance normal to the wall) as the ordinate and t_s, the time measured from the spot trigger, as the abscissa. For $m = 0$, the contours are similar to those previously published.[4] A comparison, between $m = 0$ and $m = 0.05$, of positive and negative contours of equal value of $\tilde{T}/\Delta T$ indicate that the vertical and longitudinal extents of the spot are significantly reduced by the pressure gradient. Contours in the (t_s, z) plane (not shown) indicated that the extent of the spot is also reduced in the spanwise direction.

Figure 2 shows the locations of t_{LE} and t_{TE} at five stations. To a reasonable approximation, the loci are straight lines with practically the same origin for $m = 0$ and $m = 0.05$. The slopes of these lines represent the convection velocities of LE and TE. For $m = 0$, U_{LE} is $0.83U_s$, whereas U_{TE} is $0.57U_s$, in good agreement with previous results.[7] For $m = 0.05$, the values of U_{LE}/U_s *and* U_{TE}/U_s are 0.86 and 0.66, respectively. The favorable pressure gradient increases U_{TE} by about 16%, but the 6% increase in U_{LE} is of the order of uncertainty in the measurements. The longitudinal elongation rate of the spot dL/dx_s, where $L = (U_{LE} - U_{TE})\, t_{LE}$, is decreased by 34%. Matsui[2] observed that the difference between U_{LE} and U_{TE} was due to the generation of new vortices near the rear of the spot. The favorable pressure gradient may inhibit this generation, thus accounting for the increase in U_{TE}.

The height y_s and half-width z_s of the spot can be arbitrarily defined as the maximum value of y or z reached by a particular $\tilde{T}/\Delta T$ contour in the (t_s, y) and (t_s, z) planes, respectively. It is evident from Fig. 3 that the rates of growth of the spot in both the y and z directions are affected by the pressure gradient. For $m = 0.05$, the slopes dy_s/dx_s and dz_s/dx_s are 28% and 38% smaller than for $m = 0$. The spot spreads at an overall half-angle of about 6 deg in the z direction, compared with about 9 deg for $m = 0$. On the basis of flow visualizations and the order of magnitude difference between spot growth rates in the (x, y) and (x, z) planes, Riley and Gad-el-Hak[1] argued that the main mechanism for the spanwise growth is not entrainment but rather the destabilization of the laminar flow in the vicinity of the spot, especially in the near-wall region. The present results suggest that a favorable pressure gradient would reduce this destabilization and, therefore, reduce the growth rate in the (x, z) plane more than that in the (x, y)

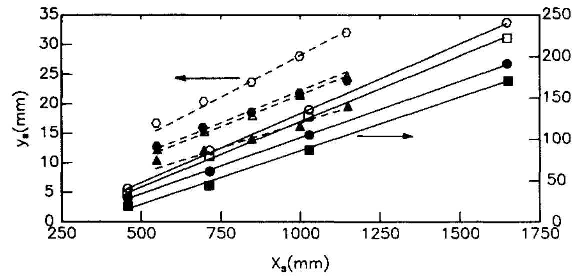

Fig. 3 Growth of the spot in the (x, y) and (x, z) planes.
a) (x, y) plane: $\tilde{T}/\Delta T = 0.03$: ○, $m = 0$; ●, $m = 0.05$; $\tilde{T}/\Delta T = 0.09$: □, $m = 0$; ■, $m = 0.05$.
b) (x, y) plane: $\tilde{T}/\Delta T = -0.03$: ⬭, $m = 0$; ⬬, $m = 0.05$; $\tilde{T}/\Delta T = -0.10$: △, $m = 0$; ▲, $m = 0.05$.

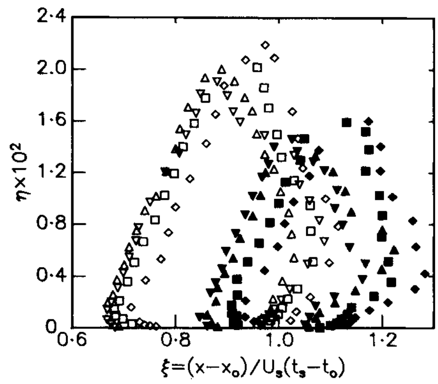

Fig. 4a Isotemperature contours in ξ, η coordinates: ◇, $x_s = 550$ mm; □, 700; △, 850; ▽, 1000 (the contour $\tilde{T}/\Delta T/0.03$ is shown; open and closed symbols are for $m = 0$ and $m = 0.05$, respectively;

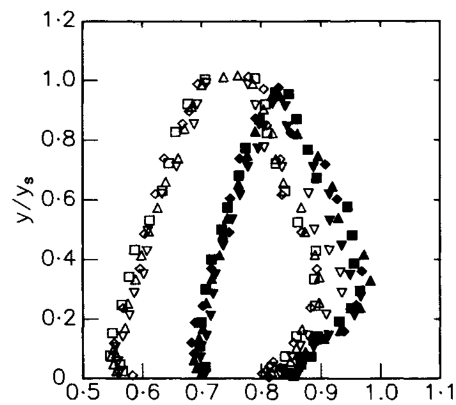

Fig. 4b Isotemperature contours in ξ_1, y/y_s coordinates (symbols same as in Fig. 4a.

plane. This contention is supported by the present 38% reduction in dz_s/dx_s, compared with a 28% reduction in dy_s/dx_s.

Sokolov et al.[8] found that the coordinates $\xi_1 = x_s/U_1 t_s$ and $\eta_1 = y/\delta$, where δ is the thickness of the hypothetical turbulent boundary layer originating near the disturbance, provided a more adequate description of the evolution of the spot than the conical similarity coordinates.[9] It was suggested that the use of δ would allow the possible influence of Reynolds number and pressure gradient to be taken into account. Here, δ, whose precise determination is awkward, has been replaced by y_s, the more easily determined height of the spot. The contours shown in Fig. 4a using conical similarity coordinates show considerable and systematic variation, especially at the leading edge. By contrast, the contours in Fig. 4b represent a marked improvement, both for $m=0$ and $m=0.05$. Similarly, when the spot half-width z_s is used as the normalizing length scale, the use of coordinates ξ_1 and z/z_s showed a significant improvement over the conical coordinates. However, the collapse was not as good as in Fig. 4b, especially near the leading edge, probably because the (t_s, z) plane measurements were made at a constant y rather than a constant y/y_s. Another reason may be the greater sensitivity of the (t_s, z) plane data, relative to data in the (t_s, y) plane, to contamination from the side walls. We estimate this contamination to become important at $x \approx 2.7$ m in this experiment; the present kinematic scaling is unlikely to apply beyond this value of x.

Conclusions and Remarks

1) With reference to zero pressure gradient, a favorable pressure gradient reduces the rate of growth in all three spatial directions. For $m=0.05$, the spanwise growth rate is reduced by nearly 38%, compared with decreases of about 35 and 28% in the longitudinal and main shear directions, respec tively.

2) The observed increase in U_{TE} for $m=0.05$ is consistent with the suggestion that the favorable pressure gradient decreases the number of structures that make up the spot.

3) Conical similarity variables do not adequately describe the overall development of the spot. The data are, however, reasonably consistent with similarity when the local height and half-width of the spot are used.

Acknowledgment

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Radial Flow Between Two Closely Placed Flat Disks

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Introduction

The flow within two closely placed flat disks can be included among those theoretical problems in fluid dynamics where, as the present paper will show, one can obtain a simple closed-form solution of the dependent variables that agrees well with the experiment. The practical aspect of the solution should not be overlooked, since disk-type devices are widely used in many engineering applications.[1] Several papers in the past dealt with the theoretical and/or experimental perspective of the problem. Kwok and Lee[2] collected and presented many experimental results. Due to the fact that velocity measurements in an extremely narrow gap (0.51 mm) are very difficult to obtain, only radial pressure distributions were presented. From the theoretical angle, most of the studies focused on integral solutions of the governing equations, with the aim to obtain the pressure variation while preserving the mean characteristics of the flow.[1-4] In a recent paper, Lee and Lin[5] attempted a differential solution, with the purpose once more to obtain the pressure distribution. Their results have shown good agreement with the experiment. However, since their linearized momentum equation cannot be solved exactly, the solution has been obtained numerically.

In the present paper, the flow within the gap is solved analytically. Simple equations for the radial static pressure distribution, the radial component of the velocity, and the global frictional coefficient are derived.

Analysis

Consider the steady, laminar, axisymmetric, nonswirling flow between the gap of two narrowly spaced flat disks shown in Fig. 1. Because the gap is small ($2h/R_o$ in the order of 1×10^{-3}), the axial velocity component of the velocity can be neglected. Under these assumptions, the equations of motion in nondimensional form are

Continuity:

$$\frac{\partial \bar{V}_r}{\partial \bar{r}} + \frac{\bar{V}_r}{\bar{r}} = 0 \tag{1}$$

R-momentum:

$$\bar{V}_r \frac{\partial \bar{V}_r}{\partial \bar{r}} = -\frac{d(\bar{\Delta P})}{d\bar{r}} + \frac{1}{\bar{R}e}\left[\xi^{\xi}\left(\frac{\partial^2 \bar{V}_r}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial \bar{V}_r}{\partial \bar{r}} - \frac{\bar{V}_r}{\bar{r}^2}\right) + \frac{\partial^2 \bar{V}_r}{\partial \bar{z}^2}\right] \tag{2}$$

where $\bar{V}_r = V_r/V_o$, $\bar{r} = \bar{r}/R_o$, $\bar{z} = z/h$, $\overline{\Delta P} = [P(r) - P_o]/\rho V_o^2$, $\xi = h/R_o$, $\bar{R}e = Re\xi$, $Re = \rho V_o h/\mu$, V_o is the inlet velocity, P_o is the inlet static pressure, ρ is the fluid density, and μ is the absolute viscosity.

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